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Optimization in Nonlinear Structural Dynamics with Reduced Order Models

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Introduction

Why account for nonlinear vibration?

- Coupled Nonlinear Dynamics/Aeroelasticity of very Flexible Aircraft
- Vibration-based MicroElectroMechanical Systems (MEMS)
- Long, Light and Flexible Blade of Wind Turbine
- High Speed Compliant Actuator
- Squeal of the Brake System

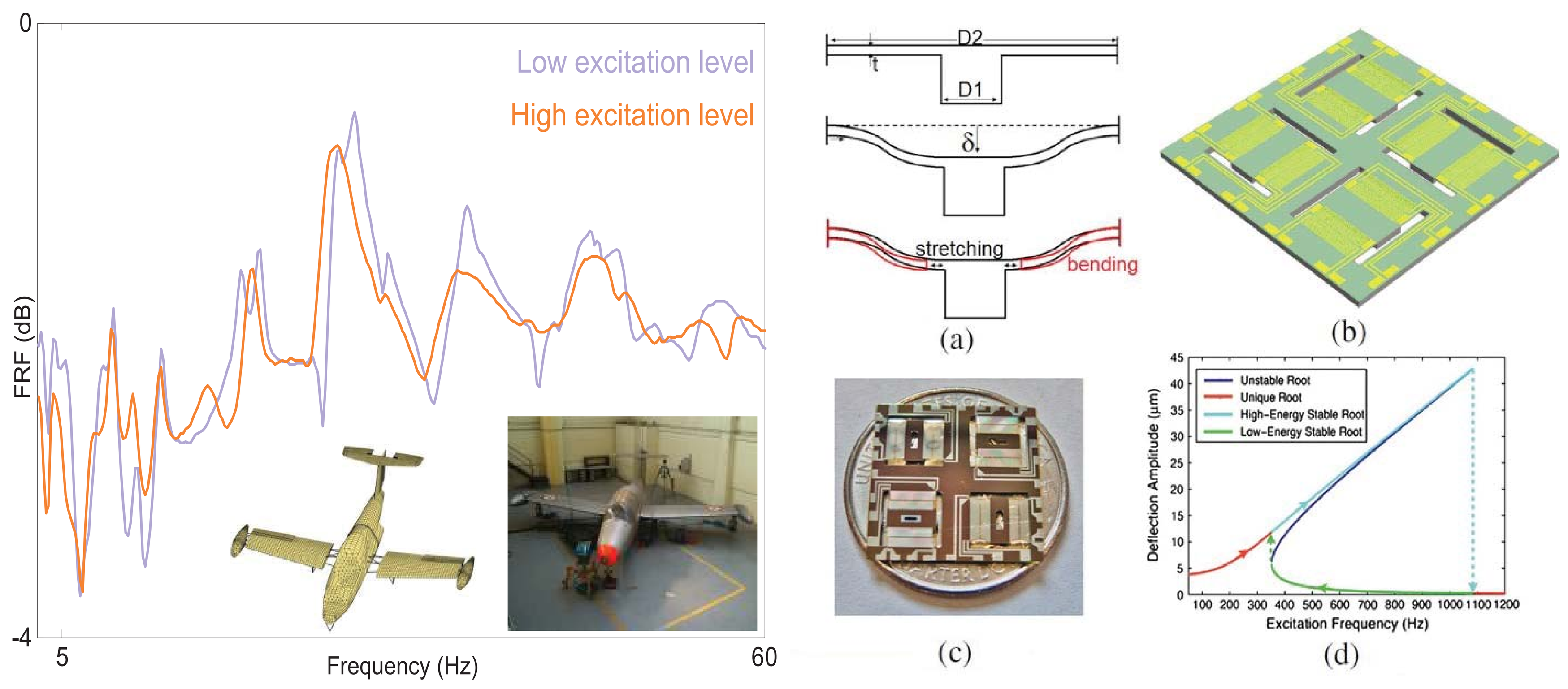

Figure 1: Nonlinear vibration in airplane and MEMS. Left: dynamic testing of an airplane presented by Gaëtan Kerschen [1]; Right: ultra-wide bandwidth piezoelectric energy harvesting device developed by Arman Hajati and Sang-Gook Kim [2].

What is the problem in optimization?

- Today's design procedures are often based on linear finite element (FE) models.
- Nonlinear structural dynamics is analyzed after a full optimization procedure.
- High computation costs of nonlinear structural dynamics are prohibitive for iterative optimization.

The goal of this PhD project

Focus on developing reduced-order models to facilitate efficient analysis and optimization.

- Eliminate the time dimension to compute the steady-state vibration efficiently.
- Reduce the spatial dimension to obtain a model with fewer degree-of-freedom.
- Do sensitivity analysis and design optimization using reduced-order models.

Method

Time-reduced models

For the time-reduced model, we consider only problems with time-harmonic excitation. These are of major relevance in machinery with rotating parts. The FE model becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}\cos\Omega t$$

in which $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ is the discretized displacement, velocity and acceleration vector, respectively. The matrices $\mathbf{M}$ and $\mathbf{C}$ represent mass and damping, and $\mathbf{g}(u)$ is a vector with the nonlinear forces, and $\mathbf{f}\cos(\Omega t)$ is the time harmonic load.
A semi-analytical method called Incremental Harmonic Balance (IHB) method is used to solve the equation of motion [3, 4]. For the incremental harmonic balance method, the governing equation becomes

$$\left(\omega^2\bar{\mathbf{M}} + \omega\bar{\mathbf{C}} + \bar{\mathbf{K}}_T(\bar{\mathbf{u}})\right)\Delta\bar{\mathbf{u}} = \bar{\mathbf{f}} - \left(\omega^2\bar{\mathbf{M}}\bar{\mathbf{u}} + \omega\bar{\mathbf{C}}\bar{\mathbf{u}} + \bar{\mathbf{g}}(\bar{\mathbf{u}})\right)$$

in which $\bar{\mathbf{u}}$ is a vector containing all coefficients of harmonics in Fourier series of $\mathbf{u}$. And $\bar{\mathbf{M}}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}_T$ are augmented matrices of mass, damping and tangent stiffness, respectively. And $\bar{\mathbf{g}}$ and $\bar{\mathbf{f}}$ are augmented vectors of elastic force and external force, respectively.

Space-reduced models (future work)

In case of transient loads, e.g. encountered in machinery start-up or with impact loads such as blasts, we need to consider the time response of the structure. The finite element model in this case becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}(t)$$

in which $\mathbf{f}(t)$ is the specific time-dependent load. Instead of computing the transient response for the full FE model, an analysis model based on nonlinear modal reduction will the applied. The space-reduced model becomes

$$\mathbf{M}_r\ddot{\mathbf{q}} + \mathbf{C}_r\dot{\mathbf{q}} + \mathbf{g}_r(\mathbf{q}) = \mathbf{f}_r(t)$$

in which $\mathbf{u}$ has been reduced to a set of generalized coordinates $\mathbf{q}$. And $\mathbf{M}_r$ and $\mathbf{C}_r$ are reduced matrices for mass and damping, respectively. And $\mathbf{g}_r$ and $\mathbf{f}_r$ are reduced vectors for elastic force and external force, respectively. The use of nonlinear modal reduction can potentially reduce computational costs by orders of magnitude.

Design optimization using reduced-order models

A general optimization problem concerning vibration is to minimize the amplitude of vibration. Based on the time-reduced models, this problem can be expressed as

$$\begin{aligned}
&\min_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} \\
\text{s.t. :}\ &\omega^2\bar{\mathbf{M}}\bar{\mathbf{u}} + \omega\bar{\mathbf{C}}\dot{\mathbf{u}} + \bar{\mathbf{g}} = \bar{\mathbf{f}}, \\
&\sum_{e=1}^{N_e}\rho_e V_e - V^* \leq 0, V_e = L_e A_e, \\
&0 < \rho_{min} \leq \rho_e \leq 1, V^* = \alpha V_0, \\
&A_e = \rho_e A_{max}, V_0 = L_e A_{max}.
\end{aligned}$$

where ρ_e are design variables, N denotes the total number of design variables, the symbol α defines the volume fraction, V_0 is the volume of the admissible design domain, and V^* is the given available volume of solid material.

Examples

Design of nonlinear beam dynamics

The structure is a doubly clamped beam with periodic load applied at the midspan. The design variable is the width $w(x)$. The objective function will be given in each example. The nonlinearity in the model arises from the midplane stretching. The axial strain ε_0 and the curvature κ are defined as

$$\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \kappa = \frac{\partial^2 w}{\partial x^2}$$

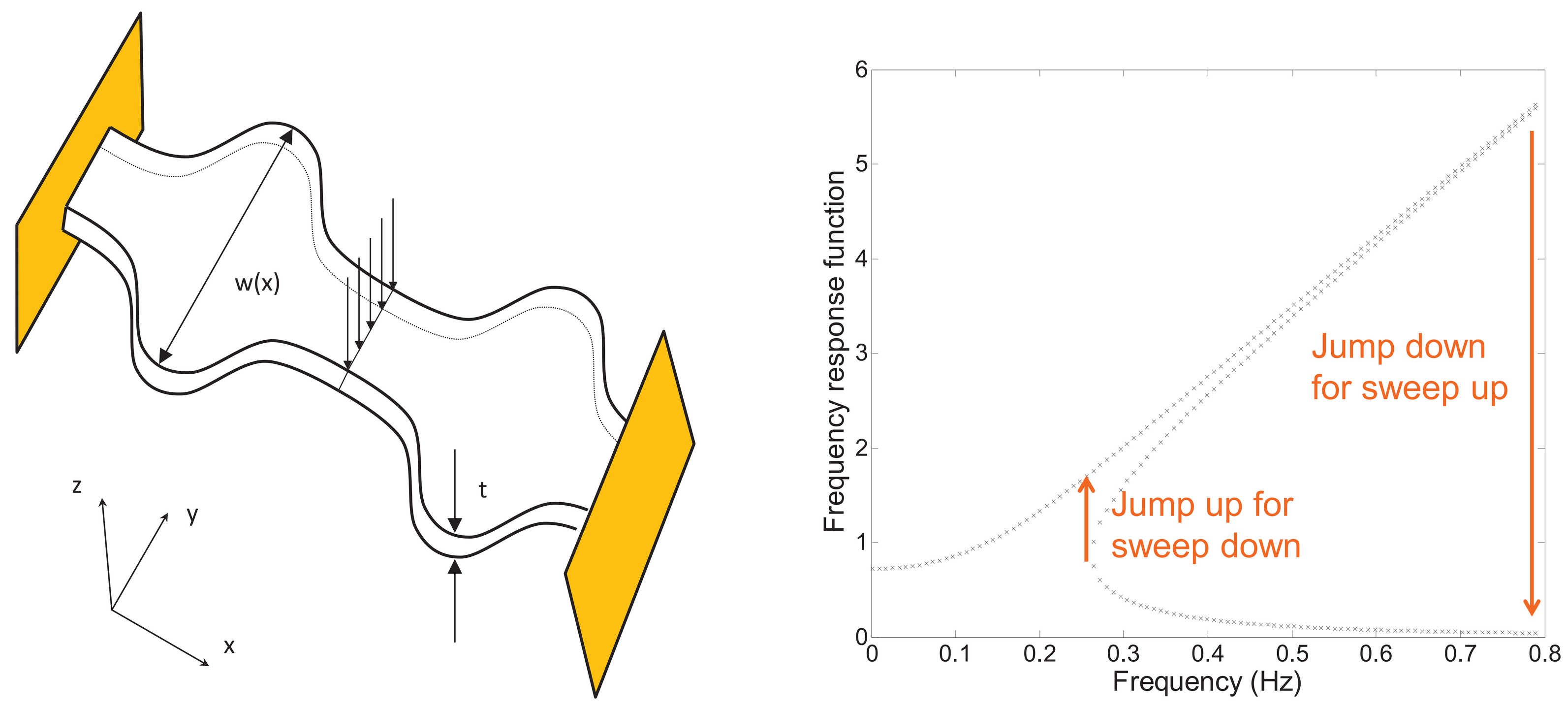

Figure 2: Schematic of the model and a typical frequency-amplitude curve.

Minimize the resonant peak

$$\min_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} = a_{i1}^2 + b_{i1}^2$$

in which a_{i1} and b_{i1} are the coefficients of the fundamental harmonic $a_{i1}\cos(\omega t) + b_{i1}\sin(\omega t)$ for the deflection at the midspan.

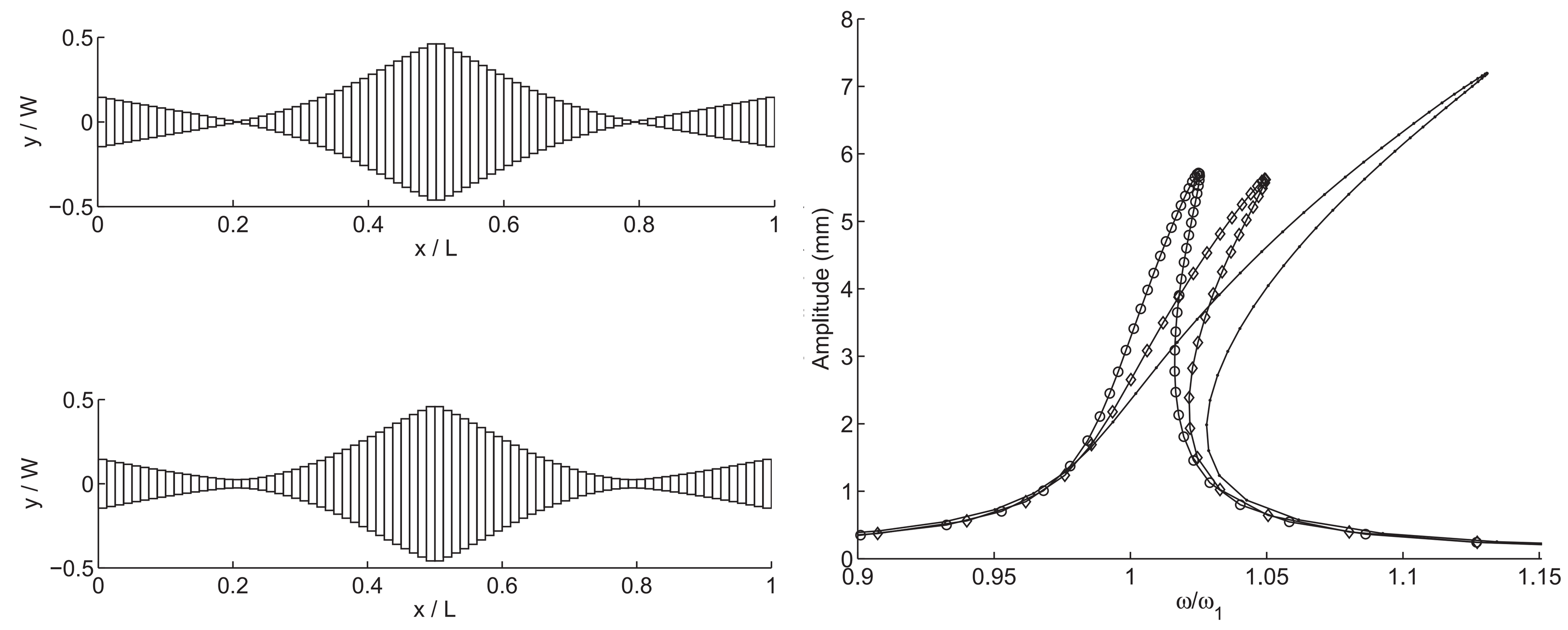

Figure 3: Optimized width for minimizing the resonant peak using linear FE model(left top) and using noninear FE model (left bottom), and nonlinear frequency-amplitude curves: circles denote optimized width using linear FE model, diamonds denote optimized width using nonlinear FE model and dots denote uniform width.

Maximize the super-harmonic resonance

$$\max_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} = a_{i3}^2 + b_{i3}^2$$

in which a_{i3} and b_{i3} are the coefficients of the third-order harmonic $a_{i3}\cos(3\omega t) + b_{i3}\sin(3\omega t)$ for the deflection at the midspan.

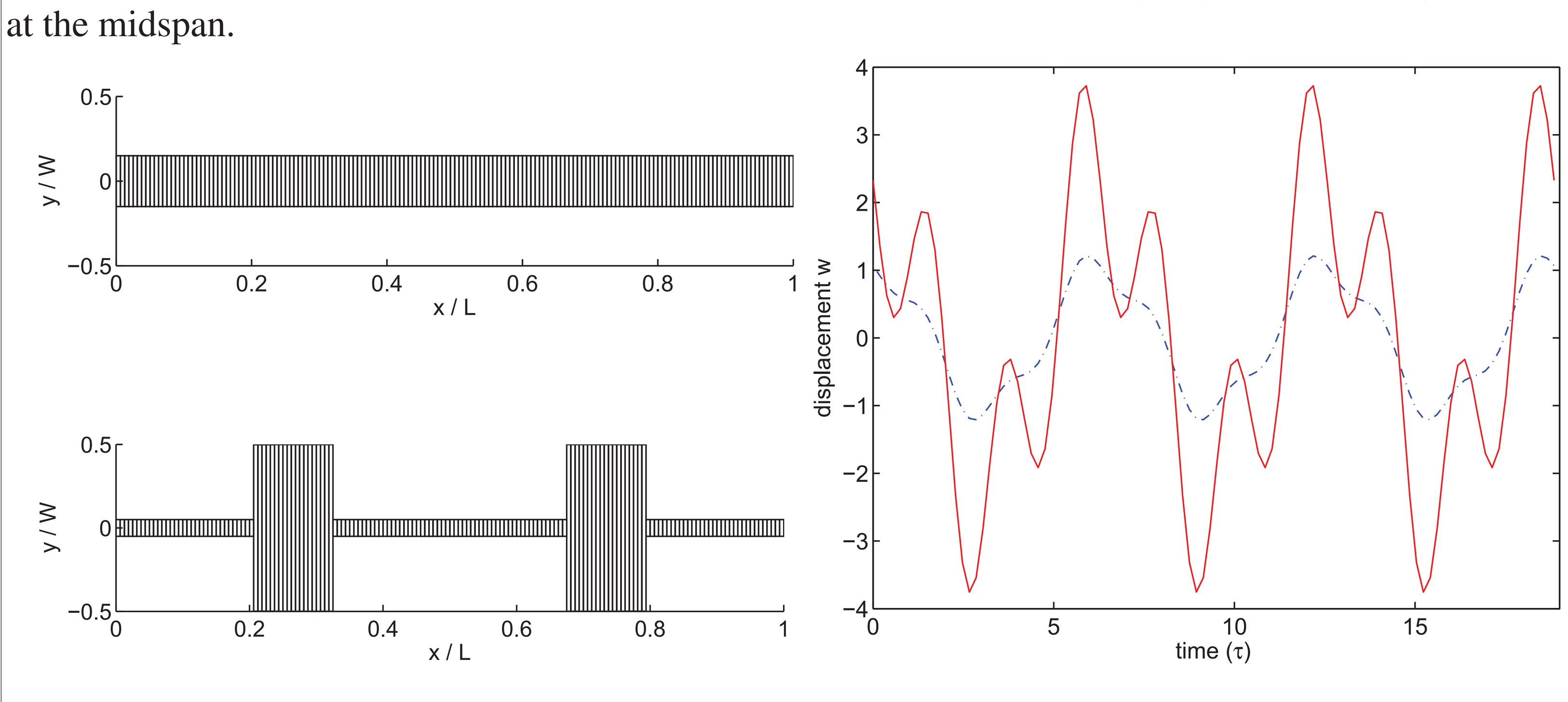

Figure 4: Optimized width for maximizing super-harmonic resonance (Left top: uniform width; Left bottom: optimized width) and the responses before optimization (dashed line) and after optimization (solid line).

Discussion

- Optimized width for minimizing the resonant peak using nonlinear FE model does not have "weak" links.
- Nonlinear structural dynamics is essential for optimizing high-order harmonics in the response.

References

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